» A History of Number Systems

In relation to Topic 11 – Solution of Quadratic Equations

Kind of number	**Invented because…**	**Weird because…**
Unary system (𝍷𝍷, 𝍸)	Basic need for counting – good for visualising amounts	Can only handle very small numbers
Roman numerals (III, XIV, LX)	Added letters that stood for large numbers	Letters have no intuitive numerical meaning
Place system with a base (23, 51, 4359)	Multiplication & division were too difficult.	A digit's position in a number changes its value
Zero (0)	How to write a number like CCI?	How can "nothing" be a number?
Fractions/decimals (½, ¾, 0.4, 0.22)	24 ÷ 6 is easy enough, but what is 24 ÷ 5?	How can there be numbers between 0 & 1?

	Negative Numbers (–x)	Complex Numbers (a + bi)
Invented because…	What is 3 – 4?	What is $\sqrt{-1}$?
Weird because…	How can you have less than nothing?	How can a number multiply by itself to get a negative?
Considered absurd until…	1700s	Still considered absurd!
Intuitive meaning	Opposite	Rotation
Multiplication cycle	1, –1, 1, –1… X, –X, X, –X…	1, i, –1, –i, 1, i, –1, –i… X, Y, –X, –Y, X, Y, –X, –Y…
Use in co-ordinates	Go backwards from origin	Rotate around the origin

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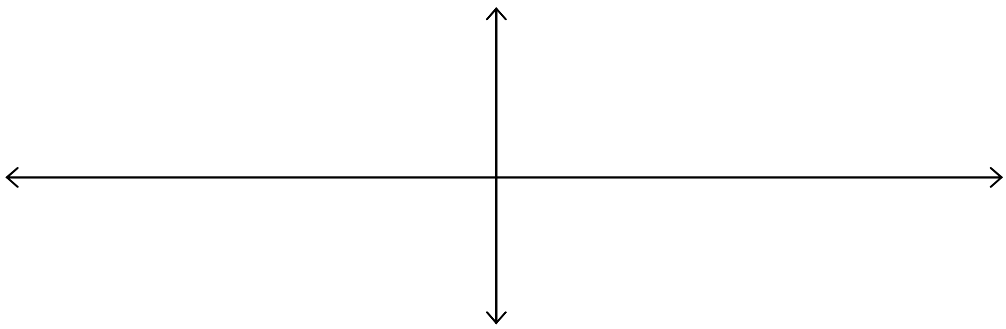
Intuitive meaning		
Multiplication cycle		
Use in co-ordinates		

» Visualising Imaginary & Complex Numbers

1. $x^2 = 9$ is short-hand for **1 × *x* × *x* = 9.**
2. What **transformation/change** "*x*", when applied twice, turns **1** into **9**?
    a. **scale** by **3**
    b. **scale** by **3** and **flip** (**flipping** represents multiplication by a **negative**)
3. What about $x^2 = -1$, which is short-hand for **1 × *x* × *x* = –1**? What **transformation** "*x*", when applied twice, turns **1** into **–1**?
    a. We can't multiply by a **positive** twice
    b. Nor can we multiply by a **negative** twice
4. What if we turned the number **line** into a number **plane**? Then we could **ROTATE** the numbers into a **new dimension** (the **Imaginary**).



5. This is positive **rotation** . We can also **rotate** negatively. That means there are two **square roots** of **–1**, **i** and **–i** (or **±i** for short).
6. This is a strange new way to think of numbers – as **two-dimensional**. Let's see if this model fits some intuitive rules that we would imagine if it was based on normal **rotation**.
    a. Three **anti-clockwise rotations** (i^3) are equal to one **clockwise rotation** (–i).
    b. Four **anti-clockwise rotations** (i^4) brings us **full circle** (1).
7. Numbers don't have to be just **Real** or **Imaginary**. They can be combinations of both. When we **rotate** , who says we need to **rotate 90**°? For example, 1 + i represents **45**°.
8. I know it's tricky. But understanding numbers in this way opens up powerful and elegant ways to do some kinds of mathematics that are otherwise very difficult (just like negative numbers did).

» Visualising Imaginary & Complex Numbers

1. $x^2 = 9$ is short-hand for ________________________.

2. What ____________________ / ______________ "x", when applied twice, turns ____ into ____?

    a. _____________ by _______

    b. _____________ by _______ and _________ (_____________ represents multiplication by a ______________)

3. What about $x^2 = -1$, which is short-hand for ________________________? What ____________________ "x", when applied twice, turns ____ into ____?

    a. We can't multiply by a __________________ twice

    b. Nor can we multiply by a __________________ twice

4. What if we turned the number ____________ into a number ______________? Then we could ___________ the numbers into a _________ ____________________ (the ___________________).

5. This is positive _____________ . We can also ________________ negatively. That means there are two ___________ ___________ of −1, _____ and _____ (or ________ for short).

6. This is a strange new way to think of numbers – as _______-_________________. Let's see if this model fits some intuitive rules that we would imagine if it was based on normal ________________ .

    a. Three _______-________________ _____________ (i^3) are equal to one ________________ _____________ (−i).

    b. Four _______-________________ _____________ (i^4) brings us __________ ____________ (1).

7. Numbers don't have to be just __________ or __________________ . They can be combinations of both. When we _____________ , who says we need to ______________ _________? For example, 1 + i represents _____________ .

8. I know it's tricky. But understanding numbers in this way opens up powerful and elegant ways to do some kinds of mathematics that are otherwise very difficult (just like negative numbers did).