Co-ordinate Geometry | Sufficiency Conditions of Quadrilaterals

To *prove* that a figure is a particular kind of quadrilateral requires the use of sufficiency conditions. Once these conditions are met, it is sufficient to prove that it is certain type of shape. Whilst all of the conditions listed below are equally valid, they do not require equal amounts of work to calculate. Some conditions are much faster to establish than others, and these should be preferred when writing proofs.

Sufficiency Condition	**Required Co-ordinate Geometry Methods**
PARALLELOGRAM	
Both pairs of opposite sides parallel	$4 \times$ gradient (each side)
Both pairs of opposite sides equal	$4 \times$ distance (each side)
Both pairs of opposite angles equal	$4 \times$ gradient (each side), then use $m = \tan\theta$ to find angles. But if you have the 4 gradients, it is faster to use the "both pairs of opposite sides parallel" condition.
Diagonals bisect each other	$2 \times$ midpoint (opposite vertices)
A pair of sides are both equal and parallel	$2 \times$ distance (opposite sides) $2 \times$ gradient (same sides)
RECTANGLE	
All angles at vertices are 90°	$4 \times$ gradient (each side); show that adjacent sides have gradients that multiply to -1
A parallelogram with one angle at a vertex 90°	$4 \times$ gradient (each side); prove that shape is a parallelogram since both pairs of opposite sides are parallel; show that one pair of adjacent sides has gradients that multiply to -1 OR $2 \times$ midpoint (opposite vertices); prove that a shape is a parallelogram since diagonals bisect each other; $2 \times$ gradient (same sides); show that one pair of adjacent sides has gradients that multiply to -1
A parallelogram with diagonals equal	$2 \times$ midpoint (opposite vertices); prove that a shape is a parallelogram since diagonals bisect each other; $2 \times$ distance (diagonals); show that distances are equal
RHOMBUS	
All sides equal	$4 \times$ distance (each side)
Diagonals bisect each other at 90°	$2 \times$ midpoint (opposite vertices); $2 \times$ gradient (diagonals); show that diagonals have gradients that multiply to -1
A parallelogram with a pair of adjacent sides equal	$2 \times$ midpoint (opposite vertices); prove that a shape is a parallelogram since diagonals bisect each other; $2 \times$ distance (adjacent sides); show that distances are equal

SQUARE	
Equal sides and one angle at a vertex equals 90°	4 × distance (each side); 2 × gradient (adjacent sides); show that one pair of adjacent sides has gradients that multiply to -1
Diagonals equal and bisect at 90°	2 × distance (diagonals); 2 × midpoint (opposite vertices); 2 × gradient (diagonals); show that diagonals have gradients that multiply to -1
KITE	
Pairs of adjacent sides are equal	4 × distance (each side)
One diagonal bisects the other diagonal at 90°	2 × gradient (diagonals); show that diagonals have gradients that multiply to -1; midpoint (bisected diagonal); line equation (non-bisected diagonal); show that midpoint satisfies line equation