# » Spheres: Volume & Surface Area

Calculating the volume and surface area of a sphere *exactly* is not an easy task – in fact, to do it precisely and mathematically requires knowledge that you do not reach until about year 11. However, the Greek mathematician Archimedes made some helpful practical observations that can help us to arrive at the formulae for calculating these values, even if the methods used to arrive at them are not precise. Archimedes observations centred around the relationship between a sphere and a cylinder that shared its radius & height. Let the sphere's radius equal r units.
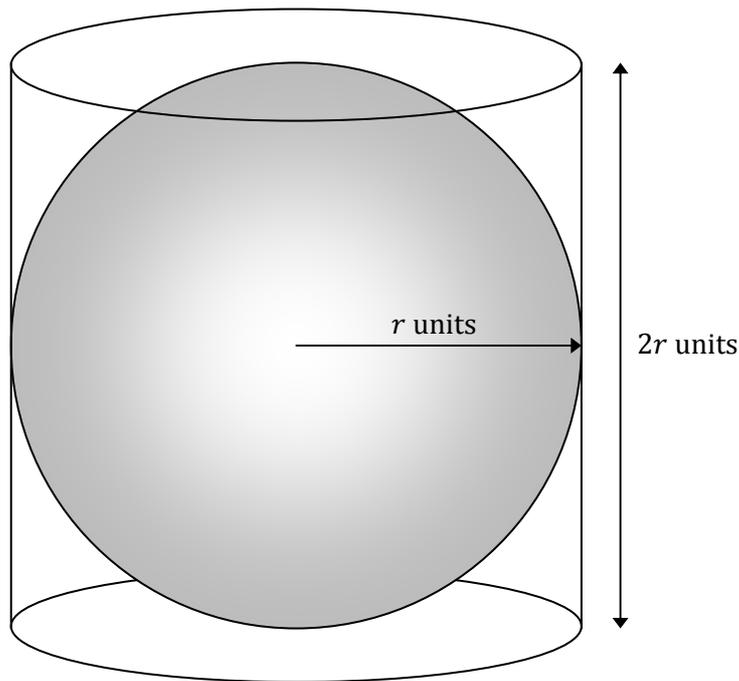
» Questions

1. Archimedes first noticed that if he took a rectangular sheet of material that wrapped exactly around the curved surface of the cylinder, the same sheet could be cut and arranged in a way that covered the entire sphere with no material wasted. Use this fact to calculate the surface area of the sphere.
2. Archimedes then filled his cylinder with water. He then completely submerged the sphere inside the filled cylinder, forcing much of the water to overflow out of the cylinder. When he removed the sphere again, he noticed that the cylinder had exactly a third of its water remaining. Use this fact to calculate the volume of a sphere.

» Solutions

1. **Archimedes first noticed that if he took a rectangular sheet of material that wrapped exactly around the curved surface of the cylinder, the same sheet could be cut and arranged in a way that covered the entire sphere with no material wasted. Use this fact to calculate the surface area of the sphere.**

Based on Archimedes' observation, we can conclude that the surface area of a sphere is equal to the curved surface area of a cylinder with the same dimensions.

$$\therefore \text{SA}_{\text{sphere}} = \text{curvedSA}_{\text{cylinder}}$$
$$= \text{circumference} \times \text{height}$$
$$= 2\pi r \times 2r$$
$$= 4\pi r^2 \text{ units}^2$$

2. **Archimedes then filled his cylinder with water. He then completely submerged the sphere inside the filled cylinder, forcing much of the water to overflow out of the cylinder. When he removed the sphere again, he noticed that the cylinder had exactly a third of its water remaining. Use this fact to calculate the volume of a sphere.**

Based on Archimedes' observation, we can conclude that the volume of a sphere is equal to two–thirds the volume of a cylinder with the same dimensions.

$$\therefore \text{V}_{\text{sphere}} = \frac{2}{3} \times \text{V}_{\text{cylinder}}$$
$$= \frac{2}{3} \times \text{base} \times \text{height}$$
$$= \frac{2}{3} \times \pi r^2 \times 2r$$
$$= \frac{4}{3}\pi r^3 \text{ units}^3$$