» Solutions: Term 4 2008, 2U Question 4

» Part (a)

Differentiate with respect to *x* and simplify:



» Part (b)

(i) Show that the volume of the cone is given by the formula $V = \frac{\pi}{3}(6h^2 - h^3)$.



(ii) Find the height of the cone so that its volume is maximized.



V is concave down, therefore the stationary point at h = 4 is a relative maximum.

Since there is only one stationary point for h > 0 and V is continuous for h > 0, the relative maximum is also the absolute maximum.

 \therefore The volume of the cone is maximized when the height is 4cm.



» Part (c) Given that $y = (1 - x)e^{x}$ (i) Show that $y' = -xe^{x}$. $y = e^{x} - xe^{x}$ $y' = e^{x} - (xe^{x} + e^{x})$ $y' = -xe^{x}$ (differentiation; product rule) $y' = -xe^{x}$ (ii) Find the co-ordinates of any stationary points and determine their nature.



(iii) Find the co-ordinates of any inflexion points.

Points of inflexion occur when y'' = 0, i.e. $-e^{x}(x + 1) = 0$



Test to verify that this is a point of inflexion (unlike x = 0 in $y = x^4$):



 $y^{\prime\prime}$ changes sign about the tested point, therefore there is a change in concavity.

 $\therefore \left(-1, \frac{2}{e}\right)$ is a point of inflexion.