» Marking Scheme: Term 2 2009, 2U Question 3

	MARKS
» Part (a)	
(i) $P = Ae^{kt}$ $\therefore \dfrac{dP}{dt} = kAe^{kt}$ $= kP$	1 for differentiation
(ii) $A = 1000$ But when $t = 10, P = 1400$. $\therefore 1400 = 1000e^{10k}$ $\frac{7}{5} = e^{10k}$	1 for substitution and simplification
$\ln\left(\frac{7}{5}\right) = 10k$ $k = \frac{1}{10}\ln 1.4$	1 for exact value of k
(iii) When $t = 20$, $P = 1000e^{20k}$ $= 1000e^{2ln1.4}$	1 for substitution
$= 1960$	1 for final answer
(iv) When $P = 3000$, $3000 = 1000e^{kt}$ $3 = e^{kt}$ $\ln 3 = kt$	1 for substitution
$t = \frac{1}{\mathrm{k}}\ln 3$ $\approx 32.65 \ldots$ $\therefore$ Population of 3000 is reached during year 1998.	1 for correct year

» Part (b)

MARKS

(i) Parabola and line intersect at $P(p, q)$.

$$nx + my = 1$$
$$x = \frac{1-my}{n}$$

1 for changing subject and solving simultaneously

Substitute into equation of parabola ($y = q$):

$$q^2 = 4a\left(\frac{1-mq}{n}\right)$$
$$nq^2 = 4a - 4amq$$
$$nq^2 + 4amq - 4a = 0$$

1 for substituting x and y for p and q

(ii) Line is tangent to parabola if discriminant of $nq^2 + 4amq - 4a$ is 0.

$$b^2 - 4ac = 0$$
$$(4am)^2 - 4(-4an) = 0$$
$$16a^2m^2 + 16an = 0$$
$$am^2 + n = 0$$

1 for substitution into discriminant

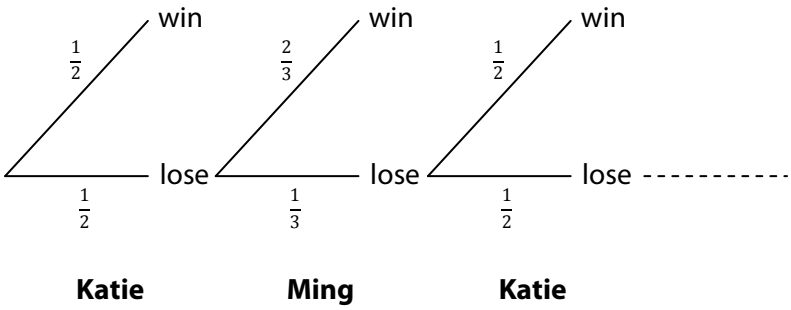
1 for simplification

» Part (c)

MARKS

(i) Katie wins by rolling a 2, 4 or 6.

Ming wins by rolling a 1, 3, 5 or 6.



$\therefore P(\text{Katie wins on throw 1}) = \frac{1}{2}$ — 1 for probability

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(ii) $P(\text{Katie wins on throw 2}) = P(\text{Katie loses}) \times P(\text{Ming loses}) \times P(\text{Katie wins})$ — ½ for reasoning

$= \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{12}$ — ½ for probability

(iii) $P(\text{Katie wins}) = \frac{1}{2} + \frac{1}{12} + \frac{1}{72} \ldots$

$= \frac{a}{1-r}$ where $a = \frac{1}{2}, r = \frac{1}{6}$ — 1 for limiting sum

$= \frac{1}{2} \div \frac{5}{6}$

$= \frac{3}{5}$ — 1 for probability