» Marking Scheme: Term 1 2009, 2U Question 1

» Part (i)

(a) $\int e^{3x} - \sec^2 2x\, dx = \frac{1}{3}e^{3x} - \frac{1}{2}\tan 2x + c$

(b) $\int \pi + \cos 3x\, dx = \pi x + \frac{1}{3}\sin 3x + c$

(c) $\int \frac{x^2+1}{x\sqrt{x}}dx = \int x^{\frac{1}{2}} + x^{\frac{-3}{2}}dx$

$= \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{-1}{2}} + c$

$= \frac{2}{3}x\sqrt{x} - \frac{2}{\sqrt{x}} + c$

2 marks each
1 per term
½ mark deducted for missing constant

» Part (ii)

(a) $\frac{1}{x-1} - \frac{1}{x+1} = \frac{x+1}{(x-1)(x+1)} - \frac{x-1}{(x-1)(x+1)}$

$= \frac{x+1-(x-1)}{x^2-1}$

$= \frac{2}{x^2-1}$

1 mark
Addition of fractions

(b) $\int_3^5 \frac{dx}{x^2-1} = \frac{1}{2}\int_3^5 \left(\frac{1}{x-1} - \frac{1}{x+1}\right) dx$

$= \frac{1}{2}[\ln(x-1) - \ln(x+1)]_3^5$

$= \frac{1}{2}[(\ln 4 - \ln 6) - (\ln 2 - \ln 4)]$

$= \frac{1}{2}(2\ln 4 - \ln 12)$

$= \ln 4 - \ln\sqrt{12}$

$= \ln\frac{2\sqrt{3}}{3}$

1 mark
Correct integral

1 mark
Simplification
½ mark deducted for decimal approximation only

» Part (iii)

$\frac{d}{dx}x\sin x = vu' + uv'$

$= \sin x + x\cos x$

1 mark
Correctly differentiated

Let $u = x, \therefore u' = 1$

$v = \sin x, \therefore v' = \cos x$

$x\sin x = \int \sin x\, dx + \int x\cos x\, dx$

$\int x\cos x\, dx = x\sin x - \int \sin x\, dx$

$\int x\cos x\, dx = x\sin x + \cos x + c$

1 mark
Re-arrangement

1 mark
Correct integral
½ mark deducted for missing constant

» Part (iv)

$$\begin{aligned}\frac{dy}{dx} &= 3x^2 - 8x + 3 \\ y &= \int (3x^2 - 8x + 3)dx \\ &= x^3 - 4x^2 + 3x + c\end{aligned}$$

1 mark
Correct integration

But the curve passes through $(0, 0)$. $\therefore c = 0$.

1 mark
Constant and subsequent factorisation of cubic

$$\begin{aligned}\therefore y &= x^3 - 4x^2 + 3x \\ &= x(x^2 - 4x + 3) \\ &= x(x - 1)(x - 3)\end{aligned}$$

$\therefore$ Curve passes through the x-axis at $x = 0, x = 1$ and $x = 3$.

1 mark
x-intercepts