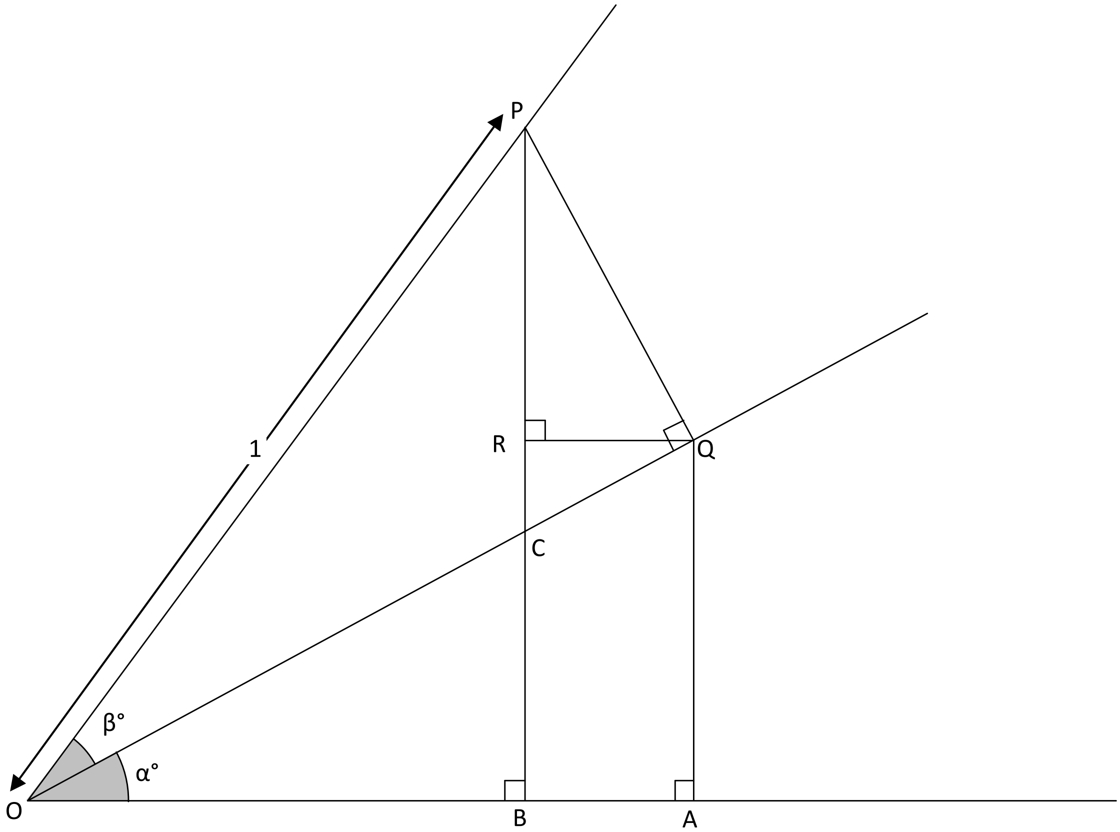
Proof for **sin(α+β)** & **cos(α+β)** identities



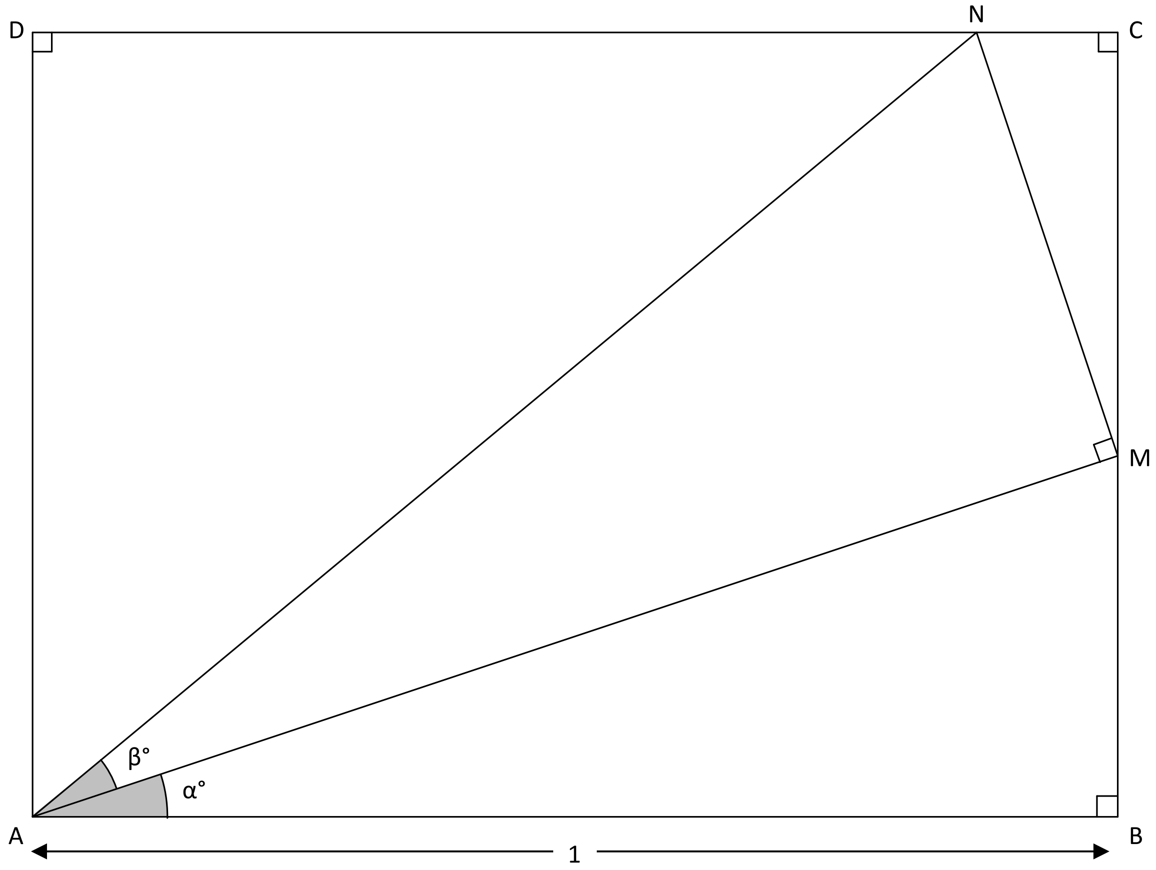
Given OP = 1, OQ⊥PQ, RQ⊥BP, AO⊥BP & AO⊥AQ
calculate the following. Find angles in terms of α & β, and distances in terms of trigonometric functions of α & β.

(i) Distance PQ
(ii) Distance OQ
(iii) Distance AQ
(iv) ∠BPQ
(v) Distance PR
(vi) Hence, find sin(α+β).
(vii) Distance QR (based on ∠BPQ & PQ)
(viii) Distance OA (based on OQ)
(ix) Hence, find cos(α+β).

Using these results, find the exact values of:

(x) sin(75°)
(xi) cos(75°)
(xii) cos(105°)
(xiii) sin(135°)
(xiv) sin(2x)
(xv) cos(2x)

Proof for **tan(α+β)** identity



Given AB = 1, AM⊥MN, and ABCD as a rectangle, calculate the following. Find angles in terms of α & β, and distances in terms of trigonometric functions of α & β.

(i) Distance BM

(ii) Distance AM

(iii) Distance MN

(iv) ∠CMN

(v) Distance CM

(vi) Distance CN

(vii) ∠AND

(viii) Hence, find tan(α+β).

(ix) Find an expression for tan(2x).

» **Answers** | sin(α+β) & cos(α+β) identities

(i) $PQ = \boldsymbol{\sin\beta}$

(ii) $OQ = \boldsymbol{\cos\beta}$

(iii) $AQ = \boldsymbol{\sin\alpha\cos\beta}$

(iv) $\angle BPQ = \boldsymbol{\alpha}°$

(v) $PR = \boldsymbol{\cos\alpha\sin\beta}$

(vi) $\sin(\alpha+\beta) = \boldsymbol{\sin\alpha\cos\beta + \cos\alpha\sin\beta}$

(vii) $QR = \boldsymbol{\sin\alpha\sin\beta}$

(viii) $OA = \boldsymbol{\cos\alpha\cos\beta}$

(ix) $\cos(\alpha+\beta) = \boldsymbol{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$

(x) $\sin(75°) = \boldsymbol{\frac{\sqrt{2}+\sqrt{6}}{4}}$

(xi) $\cos(75°) = \boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}$

(xii) $\cos(105°) = \boldsymbol{\frac{\sqrt{2}-\sqrt{6}}{4}}$

(xiii) $\sin(135°) = \boldsymbol{\frac{1}{\sqrt{2}}}$

(xiv) $\sin(2x) = \boldsymbol{2\sin x\cos x}$

(xv) $\cos(2x) = \boldsymbol{\cos^2 x - \sin^2 x}$

» **Answers** | tan(α+β) identity

(i) $BM = \boldsymbol{\tan\alpha}$

(ii) $AM = \boldsymbol{\frac{1}{\cos\alpha}}$ (can also be written as $\boldsymbol{\sec\alpha}$)

(iii) $MN = \boldsymbol{\frac{\tan\beta}{\cos\alpha}}$ (can also be written as $\boldsymbol{\sec\alpha\tan\beta}$)

(iv) $\angle CMN = \boldsymbol{\alpha}°$

(v) $CM = \boldsymbol{\tan\beta}$

(vi) $CN = \boldsymbol{\tan\alpha\tan\beta}$

(vii) $\angle AND = (\boldsymbol{\alpha+\beta})°$

(viii) $\tan(\alpha+\beta) = \boldsymbol{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$

(ix) $\tan(2x) = \boldsymbol{\frac{2\tan x}{1-\tan^2 x}}$