» The Locus of a Parabola

Yesterday we established that a parabola is the locus of points equidistant from a given point (the focus) and a given line (the directrix).

We also looked at the special case of where the focus was $(0, a)$ and the directrix was $y = -a$. The locus of such points is represented by the equation $x^2 = 4ay$. Since there is no constant term, this parabola passes through the origin – and in fact, we observed that the origin is also the parabola's vertex.

» Calculating focus & directrix

So far we have already calculated the equation of a parabola given its focus and directrix. However, we can also go in the opposite direction using the knowledge above. Note that the *focal length* of a parabola is the distance from the focus to the vertex.

1. What are the focus and directrix for each of the following parabolas? Hint: calculate a.
    a. $x^2 = 4y$
    b. $x^2 = 8y$
    c. $x^2 = 2y$
    d. $y = 4x^2$
    e. $y = x^2$
    f. $y = -2x^2$
2. What is the focal length of the parabola $y = \frac{x^2}{4}$? Where is the focus?
3. A parabola has its vertex at the point (3, 1) and focus at the point (3, 3).
    a. What is its focal length?
    b. What is the equation of its directrix?
    c. What is the parabola's equation?
4. A parabola has the line $y = -3$ as its directrix and the point (0, 1) as its focus.
    a. Where is the vertex?
    b. What is the parabola's equation?
5. Find the co-ordinates of the minimum value of $y = x^2 - 6x + 12$.
6. Find the axis of symmetry and maximum value of $y = 12 - 4x - x^2$.
7. If the focus of a parabola is $(a, 0)$ and its directrix is $x = -a$, show that its equation is $y^2 = 4ax$.