» Linear Trigonometric Equations – Worked Solutions

Unit 12 – Trigonometric Identities & Equations

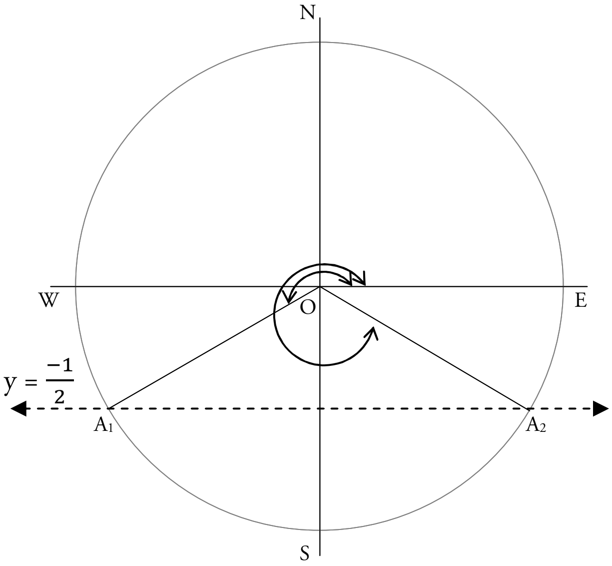
Program Items: 12(c)

Questions:
1. $2\sin A = -1$ $\{0^c \le A \le 2\pi^c\}$
2. $3\cos B - 1 = 0$ $\{|B| \le \pi^c\}$
3. $\sec^2 C = 2$ $\{0^c \le C \le 2\pi^c\}$
4. $2\tan D - \sqrt{12} = 0$ $\{|D| \le \pi^c\}$
5. $\sin^2 F\cos^2 F = \frac{1}{16}$ $\{0^c \le E \le 2\pi^c\}$

1. 2sinA = –1 {0^c≤A≤2π^c}

The first step is to have the trigonometric function by itself on the left hand side.

$$2\sin A = -1$$
$$\sin A = \frac{-1}{2}$$



This can be solved by looking at the unit circle (shown at right). The points that make up the circumference of the unit circle have the co-ordinates (cosA, sinA). In other words, if we are searching for the angle A such that $\sin A = \frac{-1}{2}$, we are searching for the points on the unit circle whose y-values are $\frac{-1}{2}$. These two points, A_1 & A_2, are marked on the unit circle at right.

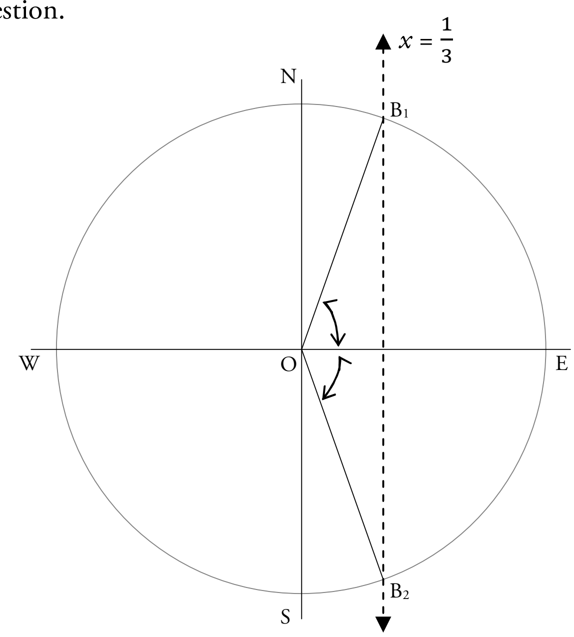
The angles that will give us this result are shown with arrows. Since $\angle A_1OW = \angle EOA_2 = 30° \left(\frac{\pi^c}{6}\right)$, we can see that:

- the smaller answer is $210° \left(\frac{7\pi^c}{6}\right)$, and
- the larger answer is $330° \left(\frac{11\pi^c}{6}\right)$.

2. 3cosB – 1 = 0 {|B|≤π^c}

The first step in answering this question is the same as the previous question.

$$3\cos B - 1 = 0$$
$$3\cos B = 1$$
$$\cos B = \frac{1}{3}$$



But from here, the question differs in two ways.

Firstly, there is no 'standard' angle that will give $\cos B = \frac{1}{3}$. This means we will have to use a calculator to evaluate the basic angle that we will then put into our solution (this can be done by typing $\cos^{-1}\left(\frac{1}{3}\right)$).

Secondly, the domain of the question is defined differently. Though it is shown here with an absolute value sign, this domain can also be expressed as $\{-\pi^c \le B \le \pi^c\}$. The way this changes the answer can again be seen on the unit circle. Since $\cos B = \frac{1}{3}$, we are searching the points on the unit circle whose x-values are $\frac{1}{3}$. B_1 & B_2 have again been marked accordingly.

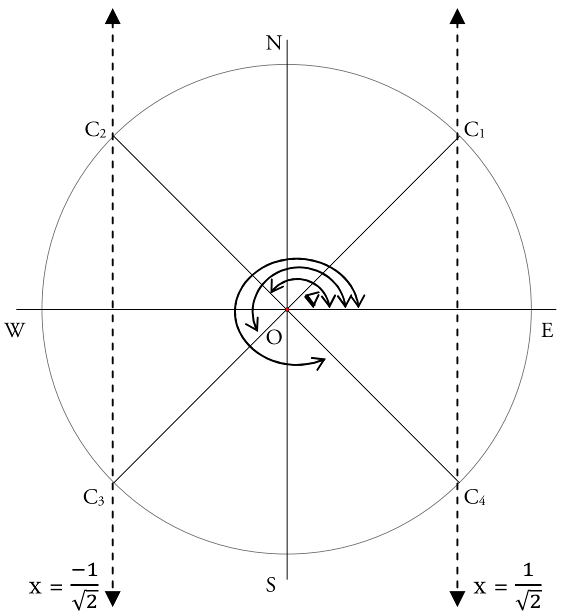
The angles that will give us this result are shown with arrows. Since $\angle EOB_1 = \angle EOB_2 = 70°32'$ (1.23^c), we can see that:

- the answer above the x-axis is $70°32'$ (1.23^c), and
- the answer below the x-axis is $-70°32'$ (-1.23^c).

3. $\sec^2 C = 2$ $\{0^c \leq C \leq 2\pi^c\}$

This question begins with the trigonometric function isolated on the left already – but this particular function is not very useful to us as we find the solution! So it needs to be simplified.

$$\sec^2 C = 2$$
$$\frac{1}{\cos^2 C} = 2$$
$$\cos^2 C = \frac{1}{2}$$
$$\cos C = \pm\frac{1}{\sqrt{2}}$$



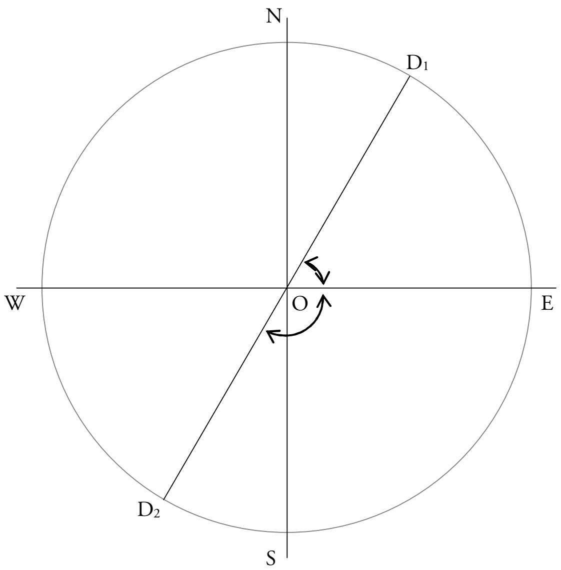
The basic angle that solves this trigonometric equation is 45° $\left(\frac{\pi^c}{4}\right)$. This can be seen on the unit circle at right. However, instead of producing two solutions, the equation above produces four solutions because of the ±. Thinking in terms of the "ASTC" acronym and the four quadrants, the effect of the ± is to produce a solution in every single quadrant. The four angles that will give us the desired result are shown with arrows. Since $\angle EOC_1 = \angle WOC_2 = \angle WOC_3 = \angle EOC_4 = 45° \left(\frac{\pi^c}{4}\right)$, we can see that:

- the first answer is 45° $\left(\frac{\pi^c}{4}\right)$,
- the second answer is 135° $\left(\frac{3\pi^c}{4}\right)$,
- the third answer is 225° $\left(\frac{5\pi^c}{4}\right)$, and
- the fourth answer is 315° $\left(\frac{7\pi^c}{4}\right)$.

4. $2\tan D - \sqrt{12} = 0$ $\{|D| \leq \pi^c\}$

We begin by simplifying:

$$2\tan D - \sqrt{12} = 0$$
$$2\tan D = \sqrt{12}$$
$$2\tan D = 2\sqrt{3}$$
$$\tan D = \sqrt{3}$$



Remember that tan refers to the ratio between sin and cos – in the context of the unit circle, this means the gradient. We can see this on the unit circle on the right, where the gradient of intervals D_1O and D_2O are both equal to $\sqrt{3}$.

The angles that will give us this result are shown with arrows. Since $\angle EOD_1 = \angle WOD_2 = 60° \left(\frac{\pi^c}{3}\right)$, we can see that:

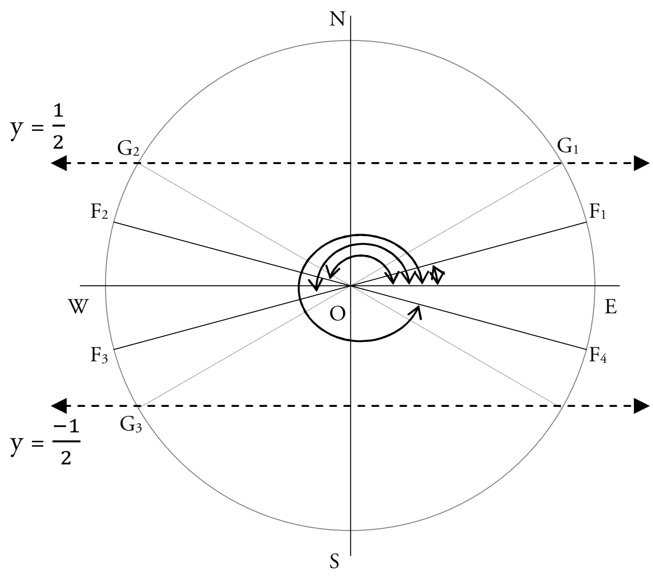
- the answer above the x-axis is 60° $\left(\frac{\pi^c}{3}\right)$, and
- the answer below the x-axis is –120° $\left(\frac{-2\pi^c}{3}\right)$.

5. $\sin^2F\cos^2F = \frac{1}{16}$ $\{0^c \leq F \leq 2\pi^c\}$

This question is the only in the set to require the use of a true trigonometric identity:

$$\sin^2F\cos^2F = \frac{1}{16}$$
$$\sin F\cos F = \pm\frac{1}{4}$$
$$2\sin F\cos F = \pm\frac{1}{2}$$
$$\sin(2F) = \pm\frac{1}{2}$$

Let G=2F $\quad \sin G = \pm\frac{1}{2}$



The identity used in this question is that 2sinFcosF = sin(2F). To find F, we must first find G, which we can then halve to find the solutions we need. The basic angle of G is 30° $\left(\frac{\pi^c}{6}\right)$, which means that the basic angle of F will be 15° $\left(\frac{\pi^c}{12}\right)$. The solutions for G are shown as G_1 through G_4, while the solutions for F are shown as F_1 through F_4.

Since $\angle EOF_1 = \angle WOF_2 = \angle WOF_3 = \angle EOF_4 = 15° \left(\frac{\pi^c}{12}\right)$, we can see that:

- the first answer is 15° $\left(\frac{\pi^c}{12}\right)$,
- the second answer is 165° $\left(\frac{11\pi^c}{12}\right)$,
- the third answer is 195° $\left(\frac{13\pi^c}{12}\right)$, and
- the fourth answer is 345° $\left(\frac{23\pi^c}{12}\right)$.