1. \( \lambda = r \theta \)
   \( 15 \text{ cm} = 8 \theta \)
   \( \theta = \frac{15}{8} \)
   \( \approx \frac{15}{8} \times \frac{180^\circ}{\pi} \)
   \( \approx 107^\circ 26' \) (nearest minute)

2. \( \lambda = r \theta \)
   \( 15 \times \left( 70 \times \frac{\pi \text{ cm}}{180} \right) \)
   \( \approx 18.33 \text{ cm} \) (2dp)

3. \( \lambda = r \theta \)
   \( = 12 \times \left( 100 \times \frac{\pi \text{ cm}}{180} \right) \)
   \( = \frac{20\pi}{3} \text{ cm} \)

4. (a) \( A = \frac{1}{2} r^2 \theta \)
   \( 240 = \frac{1}{2} \times 20^2 \times \theta \)
   \( \theta = \frac{240}{200} \)
   \( = 1.2^\circ \)
   \( \approx 68^\circ 45' \) (nearest minute)

   (b) \( \lambda = r \theta \)
   \( = 20 \times \frac{6}{5} \)
   \( = 24 \text{ cm} \)

   (c) \( c^2 = a^2 + b^2 - 2ab \cos C \)
   \( = 2(20^2) - 2(20^2) \cos \left( \frac{6}{5} \right) \)
   \( c^2 = 800 \left( 1 - \cos \left( \frac{6}{5} \right) \right) \)
   \( c \approx 22.59 \text{ cm} \) (2dp)

*Textbook incorrect*
5. Let midpoint of PQ be M.
   \[ LMOQ = \tan^{-1}\left(\frac{12}{5}\right) \]
   \[ LPOQ = 2\tan^{-1}\left(\frac{12}{5}\right) \]
   \[ \text{Arc } PQ = r \theta \]
   \[ = 13 \times 2\tan^{-1}\left(\frac{12}{5}\right) \]
   \[ = 26 \tan^{-1}\left(\frac{12}{5}\right) \]
   \[ \approx 30.58 \text{ cm (2dp)} \]

6. \[ \alpha = \cos^{-1}\left(\frac{3}{8}\right) \]
   \[ l = r \theta \]
   \[ = 5 \times 2\cos^{-1}\left(\frac{5}{8}\right) \]
   \[ = 10 \cos^{-1}\left(\frac{5}{8}\right) \]
   \[ \text{Major arc} = 2\pi r - l \]
   \[ = 10\pi - 10\cos^{-1}\left(\frac{5}{8}\right) \]
   \[ \approx 22.46 \text{ cm (2dp)} \]
   *Textbook 1 dp

7. (a) \[ \cos \angle AOB = \frac{a^2 + b^2 - c^2}{2ab} \]
   \[ = \frac{2(10^2) - 16^2}{2(10^2)} \]
   \[ \cos \angle AOB = \frac{-7}{25} \]
   \[ \angle AOB = \cos^{-1}\left(\frac{-7}{25}\right) \]
   \[ \approx 1.85^\circ \text{ (2dp)} \]
   \[ \approx 106^\circ 16' \text{ (nearest minute)} \]
7. Continued.
(b) \( L = r \theta \)
\[ = 10 \cos^{-1}\left(\frac{-7}{25}\right) \]
\[ \approx 18.55 \text{ cm (2dp)} \]

(c) \( A = \frac{1}{2} r^2 (\theta - \sin \theta) \)
\[ = 50 (\theta - \sin \theta) \]
\[ \approx 44.73 \text{ cm}^2 \ (2dp) \]

8.

(a) \( \theta = \frac{16}{60} \times 2\pi \)
\[ = \frac{8 \pi}{15} \]
\( L = r \theta \)
\[ = 20 \times \frac{8 \pi}{15} \]
\[ = \frac{32}{3} \pi \text{ cm} \]
\[ \approx 33.51 \text{ cm (2dp)} \]

(b) \( c^2 = a^2 + b^2 - 2ab \cos C \)
\[ = 2(20^2) - 2(20^2) \cos \left(\frac{8 \pi}{15}\right) \]
\( c^2 = 800 (1 - \cos \left(\frac{8 \pi}{15}\right)) \)
\[ c \approx 29.73 \text{ cm (2dp)} \]

* Textbook incorrect
9. \( \theta = \frac{\pi}{3} \)

\[ r = 15 \text{cm} \]

Arc \( AB = r \theta \)

\[ = 5\pi \text{ cm} \]

Chord \( AB = r \sqrt{2(1 - \cos \theta)} \)

\[ = 15 \sqrt{2 \left(1 - \frac{1}{2}\right)} \]

\[ = 15 \sqrt{1} \]

\[ = 15 \text{cm} \]

\[ \therefore \text{Difference} = 5\pi - 15 \text{ cm} \]

10. Remaining \( \theta = 330^\circ \)

\[ = \frac{11\pi}{6} \]

\[ \therefore A = \frac{1}{2} r^2 \theta \]

\[ = \frac{1}{2} \times 3^2 \times \frac{11\pi}{6} \]

\[ = \frac{33\pi}{4} \text{ cm}^2 \]

11. \( \theta = 140^\circ \)

\[ = \frac{7\pi}{9} \]

Arc \( = r \theta \)

\[ = \frac{112\pi}{9} \text{ cm} \]

Chord \( = r \sqrt{2(1 - \cos \theta)} \)

\[ = 16 \sqrt{2(1 - \cos \left(\frac{7\pi}{9}\right))} \]

\[ \approx 30.07 \text{ cm} \ (2dp) \]

Difference \( \approx 9.03 \text{ cm} \ (2dp) \)

Textbook: 0\ dp
12. (a) Shaded \( A = \frac{1}{2} R^2 \theta - \frac{1}{2} r^2 \theta \)
\[= \frac{\theta}{2} (R^2 - r^2)\]
\[= \frac{\pi}{8} (225 - 100)\]
\[= \frac{125\pi}{8} \text{ cm}^2\]

(b) Perimeter = \( r \theta + R \theta + 2 (R - r) \)
\[= \frac{\pi}{4} (10 + 15) + 2 (5)\]
\[= 10 + \frac{25\pi}{4} \text{ cm} \]

13. \( 40 \text{ minutes} = \frac{2}{3} \times 2\pi \text{ c} \)
\[= \frac{4\pi}{3} \text{ c} \]

Area = \( \frac{1}{2} r^2 \theta \)
\[= \frac{1}{2} (3^2) \times \frac{4\pi}{3}\]
\[= 6\pi \text{ cm}^2\]

14. Since \( AB = r \), \( \triangle AOB \) is equilateral.

\[\therefore \angle AOB = \frac{\pi}{3}\]

\[\therefore \text{Minor Segment} = \frac{1}{2} r^2 (\theta - \sin \theta)\]
\[= \frac{1}{2} r^2 \left( \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)\]

\[\text{Major Segment} = \frac{1}{2} r^2 \left( \frac{5\pi}{3} + \frac{\sqrt{3}}{2} \right)\]

\[\therefore \text{Minor} : \text{Major} = \frac{\pi}{3} - \frac{\sqrt{3}}{2} : \frac{5\pi}{3} + \frac{\sqrt{3}}{2}\]
\[2\pi - 3\sqrt{3} : 10\pi + 3\sqrt{3}\]
15.

\[ \cos \theta = \frac{10}{20} \]
\[ \theta = \frac{\pi}{3} \]
\[ \phi = 2\pi - 2\left(\frac{\pi}{3}\right) \]
\[ = \frac{4\pi}{3} \]

\[ : A_{\text{segment}} = \frac{1}{2} r^2 (\theta - \sin \theta) \]
\[ = \frac{1}{2} (20^2) \left( \frac{4\pi}{3} - \sin \left( \frac{4\pi}{3} \right) \right) \]
\[ = 200 \left( \frac{4\pi}{3} + \frac{\sqrt{3}}{2} \right) \]
\[ = \frac{800\pi}{3} + 100\sqrt{3} \]
\[ \approx 1010.96 \text{ m}^2 \ (2dp) \quad \text{* Textbook 0 dp} \]

16.

\[ \text{Circumference} = l \]
\[ = r\theta \]
\[ = 10 \times 160 \times \frac{\pi}{180} \]
\[ = \frac{80\pi}{9} \text{ cm} \quad (\approx 27.93 \text{ cm}, \ 2dp) \]

\[ \star \text{Textbook approximate} \]
17. \( \theta = 30^\circ \)

(a) \( p = 2r + 1 \)

\[
= \frac{\pi}{6} 
= 50 + 25\left(\frac{\pi}{6}\right) 
\approx 63.09 \text{ m} \quad (2dp) \quad \text{* Textbook 0 dp}
\]

(b) \( A = \frac{1}{2} r^2 \theta \)

\[
= \frac{1}{2} (25^2) \frac{\pi}{6} 
= \frac{625\pi}{12} 
\approx 163.62 \text{ m}^2 \quad (2dp) \quad \text{* Textbook 1 dp}
\]

18.

(a) \( l = r \theta \)

\[
= 40 \times 25 \times \frac{\pi}{180} 
= \frac{50\pi}{9} \text{ cm} 
\approx 17.45 \text{ cm} \quad (2dp)
\]

(b) \( c = r \sqrt{2(1-\cos\theta)} \)

\[
= 40\sqrt{2(1-\cos\left(\frac{5\pi}{36}\right))} 
\approx 17.32 \text{ cm} \quad (2dp)
\]
19. \[ \cos \theta = \frac{a^2 + b^2 - c^2}{2ab} \]
\[ = \frac{10^2 + 16^2 - 10^2}{2(10 \times 16)} \]
\[ = \frac{16^2}{2(10 \times 16)} \]
\[ = \frac{16}{20} \]
\[ \cos \theta = \frac{4}{5} \]
\[ \theta = \cos^{-1} \left( \frac{4}{5} \right) \]

Overlap area = \[ 2 \times \text{Area}_{\text{segment}} \]
\[ = 2 \times \frac{1}{2} r^2 \left( 2\theta - \sin(2\theta) \right) \]
\[ = 100 \left( 2\cos^{-1}(\frac{4}{5}) - \sin \left[ 2\cos^{-1}(\frac{4}{5}) \right] \right) \]
\[ \approx 32.70 \text{ cm}^2 \, (2 \text{ dp}) \]
* Textbook incorrect

20. \[ A_\Delta = \frac{1}{2} ab \sin C \]
\[ = \frac{1}{2} \left( 10^2 \right) \left( \sin \frac{\pi}{3} \right) \]
\[ = 50 \times \frac{\sqrt{3}}{2} \]
\[ = 25\sqrt{3} \text{ cm}^2 \]

\[ A_{\text{Sector}} = \frac{1}{2} r^2 \theta \]
\[ = \frac{1}{2} \times 5^2 \times \frac{\pi}{3} \]
\[ = \frac{25\pi}{6} \text{ cm}^2 \]

\[ A_{\text{Shaded}} = A_\Delta - 3A_{\text{Sector}} \]
\[ = 25\sqrt{3} - \frac{25\pi}{2} \]
\[ \approx 4.031 \text{ cm}^2 \, (3 \text{ dp}) \]
* Textbook incorrect
EXERCISES 17(b)

1. Calculate, in degrees and minutes, the magnitude of the angle subtended at the centre of a circle of radius length 8 cm by an arc whose length is 15 cm.

2. Find the length of an arc of a circle of radius 15 cm if the arc subtends an angle of 70° at the centre.

3. An arc of a circle subtends an angle of 100° at the centre. If the radius length of the circle is 12 cm, calculate the arc length in terms of π.

4. The area of a sector OAB of a circle, centre O, radius length 20 cm is 240 cm². Calculate
(a) the magnitude of the angle AOB,
(b) the length of arc AB,
(c) the length of the chord AB.

5. A chord PQ, 24 cm long, is 5 cm from the centre of the circle. Calculate the length of the arc PQ.

6. A point P is 8 cm distant from the centre of a circle of radius length 5 cm. Find the length of the major arc between the points of contact of the tangents drawn from P to the circle.

7. A chord AB of a circle with centre O has length 16 cm. If the radius of the circle is 10 cm, calculate
(a) the magnitude of angle AOB,
(b) the length of the minor arc AB,
(c) the area of the minor segment formed by the chord AB.

8. The minute hand of a clock is 20 cm long. Calculate
(a) the arc length along which the tip of the hand travels in 16 min,
(b) the shortest distance between the initial and final positions of the tip of the hand.

9. An arc AB subtends an angle of 60° at the centre of a circle of radius 15 cm. Calculate the difference in the lengths of the chord AB and the minor arc AB in terms of π.

10. From a circular piece of metal 6 cm in diameter, a sector of angle 30° is removed. Find the area remaining. Express your answer in terms of π.

11. A chord subtends an angle of 140° at the centre of a circle of radius 16 cm. Find the difference in length between the chord and the arc.

12. Calculate, in terms of π, (a) the area of the shaded region between the two circular arcs, centre O and (b) the perimeter of the shaded region.

13. The minute hand of a clock is 3 cm in length. What area is swept out by the hand in an interval of 40 min? Express your answer in terms of π.

14. A circular metal plate is cut into two segments along a chord equal in length to the radius. What is the ratio of the areas of the two segments?

15. A sheep, grazing in a paddock, is tethered to a stake by a rope 20 m long. If the stake is 10 m from a fence, find the area over which the sheep can graze.

16. A sector of angle 160° is cut out of a circular piece of thin cardboard of radius 10 cm. The cut edges of the sector are brought together to form a cone. Find the circumference of the circular base of the cone.

17. Find (a) the perimeter (b) the area of a piece of land in the shape of a circular sector of radius 25 metres and angle 30°.

18. A pendulum 40 cm long swings through an angle of 25°. Find (a) the length of arc (b) the shortest distance between the extreme positions of the bob.

19. Two circles, each of radius length 10 cm, have their centres 16 cm apart. Calculate the area common to each circle.

20. Three circles are drawn with radius length 5 cm. Their centres are at the vertices of an equilateral triangle whose sides are of length 10 cm. Calculate the area between the circles.