» Quadratic Theory: Revision

Time was devoted in Year 9 to finding the solutions of quadratic equations. In this topic, we will go deeper in our understanding of quadratics by looking at the following points:

a. Solve equations that are not quadratics, but can be *reduced to quadratic form*
b. Solve and graph *quadratic inequalities*
c. Understand the significance of the *discriminant*
d. Express *identical* quadratics in *different forms*
e. Construct a quadratic equation *based on its roots*
f. Demonstrate the relationship between *roots* and *co-efficients* in a quadratic equation

Before we can progress to do any of that, let's make sure we have the basics down first.

» Revision: Solving Quadratics

1. Produce the factorisations of the following quadratic equations, and hence find their solutions.
    a. $6x^2 + x - 2 = 0$
    b. $-5x^2 + 9x - 4 = 0$
    c. $-7x^2 + 23x - 6 = 0$
    d. $4x^2 - x - 18 = 0$
    e. $12x^2 + 8x - 15 = 0$

2. Find the solutions of the following quadratic equations, using one of the three methods for solving quadratics (factorising, completing the square or quadratic formula). Then, equate these results with the roots provided to find the value of the unknown constant.
    a. 3 and $-1\frac{1}{2}$ are the roots of $2x^2 - ax - a^2$. Find the value of a.
    b. 12 and $1\frac{3}{5}$ are the roots of $-5x^2 + 17bx - 6b^2$. Find the value of b.
    c. $\frac{1}{2}$ and $1\frac{1}{2}$ are the roots of $4cx^2 - 12x - c^2x + 3c$. Find the value of c.
    d. $\frac{1}{3}$ and 2 are the roots of $3dx^2 - 2d^2x - 3x + 2d$. Find the value of d.

» Answers

1. First, I provide the relevant factorisation; then I write the actual solutions.
   a. $(2x-1)(3x+2); x = \frac{1}{2}$ & $\frac{-2}{3}$
   b. $(x-1)(4-5x); x = 1$ & $\frac{4}{5}$
   c. $(3-x)(7x-2); x = 3$ & $\frac{2}{7}$
   d. $(x+2)(4x-9); x = -2$ & $\frac{9}{4}$
   e. $(2x+3)(6x-5); x = \frac{-3}{2}$ & $\frac{5}{6}$

2. After giving the factorisation, I provide each equation's roots and hence the value of the unknown.
   a. $(x-a)(2x+a); x = a$ & $\frac{-a}{2}$. $\therefore a = 3$.
   b. $(x-3b)(2b-5x); x = 3b$ & $\frac{-2b}{5}$. $\therefore b = 4$.
   c. $(4x-c)(cx-3); x = \frac{c}{4}$ & $\frac{3}{c}$. $\therefore c = 2$.
   d. $(3x-2d)(dx-1); x = \frac{2d}{3}$ & $\frac{1}{d}$. $\therefore d = 3$.