1. A man finds that he is late for work on 10% of occasions if he is on time the previous day, and on 20% of occasions if he is late the previous day. If he was on time on Monday, what is the probability that he will be on time on Wednesday? Illustrate with a tree diagram.

2. To gain a driver's licence in NSW, both a written test and a practical driving test must be passed. Statistics show that 70% pass the written test on the first attempt, while 90% of those who sit the test a second time will pass. 60% pass their first practical test and 80% pass their second practical test. Suppose the written and practical test are independent.
   Calculate, as a percentage, the probability of:
   a. Passing the written test on the second attempt
   b. Passing the written test after no more than two attempts
   c. Requiring a third written test
   d. Passing the practical test on the second attempt
   e. Receiving a licence after two written tests and one practical test

3. A hand of three cards is dealt from a pack of 52 playing cards.
   Calculate, as a fraction, the probability of:
   a. Receiving the ace of hearts
   b. All three cards being spades
   c. Being dealt at least one diamond

4. An urn contains $w$ white discs and $b$ black discs. The probability of randomly selecting 2 white discs in a row, without replacement, is $\frac{1}{3}$.
   a. Write an equation that shows the relationship between $w$ and $b$.  
   b. The probability of randomly selecting 3 white discs in a row, without replacement, is $\frac{1}{6}$.
      Find the number of white discs in the urn.
A man finds that he is late for work on 10% of occasions if he is on time the previous day, and on 20% of occasions if he is late the previous day. If he was on time on Monday, what is the probability that he will be on time on Wednesday? Illustrate with a tree diagram.

\[
P(\text{Wednesday on time}) = P(\text{on time, on time}) + P(\text{late, on time})
\]
\[
= (90\%)^2 + (10\% \times 80\%)
\]
\[
= 81\% + 8\%
\]
\[
= 89\%
\]

To gain a driver’s licence in NSW, both a written test and a practical driving test must be passed. Statistics show that 70% pass the written test on the first attempt, while 90% of those who sit the test a second time will pass. 60% pass their first practical test and 80% pass their second practical test. Suppose the written and practical test are independent.

a. \[ P(\text{pass written test, 2nd attempt}) = P(\text{fail 1st written test}) \times P(\text{pass 2nd written test}) \]
\[ = 30\% \times 90\% \]
\[ = 27\% \]

b. \[ P(\text{pass written test, } \leq 2 \text{ attempts}) = P(\text{pass 1st attempt}) + P(\text{pass 2nd attempt}) \]
\[ = 70\% + 27\% \]
\[ = 97\% \]

c. \[ P(\text{require 3rd written test}) = 1 - P(\text{pass written test, } \leq 2 \text{ attempts}) \]
\[ = 3\% \]

d. \[ P(\text{pass practical test, 2nd attempt}) = P(\text{fail 1st practical}) \times P(\text{pass 2nd practical}) \]
\[ = 40\% \times 80\% \]
\[ = 32\% \]

e. \[ P(\text{pass 2nd written, 1st practical}) = 27\% \times 60\% \]
\[ = 16.2\% \]
3. A hand of three cards is dealt from a pack of 52 playing cards.

a. Receiving the ace of hearts

\[
P(\text{ace of hearts}) = P(\text{received 1st}) + P(\text{received 2nd}) + P(\text{received 3rd})
\]

\[
= \frac{1}{52} + \left( \frac{51}{52} \times \frac{1}{51} \right) + \left( \frac{51}{52} \times \frac{50}{51} \times \frac{1}{50} \right)
\]

\[
= \frac{1}{52} + \frac{1}{52} + \frac{1}{52}
\]

\[
= \frac{3}{52}
\]

b. All three cards being spades

\[
P(3 \text{ spades}) = P(1\text{st spade}) \times P(2\text{nd spade}) \times P(3\text{rd spade})
\]

\[
= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}
\]

\[
= \frac{11}{850}
\]

c. Being dealt at least one diamond

\[
P(\geq 1 \text{ diamond}) = 1 - P(\text{no diamonds})
\]

\[
= 1 - \left( \frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} \right)
\]

\[
= \frac{997}{1700}
\]
4. An urn contains $w$ white discs and $b$ black discs. The probability of randomly selecting 2 white discs in a row, without replacement, is $\frac{1}{3}$.

a. Write an equation that shows the relationship between $w$ and $b$.

$$P(2 \text{ white}) = P(1\text{st disc is white}) \times P(2\text{nd disc is white})$$

$$\frac{1}{3} = \frac{w}{b+w} \times \frac{w-1}{b+w-1}$$

$$3w(w-1) = (b+w)(b+w-1)$$

$$3w^2 - 3w = b^2 + bw - b + bw + w^2 - w$$

$$2w^2 - 2w = b^2 + 2bw - b \quad \cdots \quad \cdots \quad \text{(1)}$$

b. The probability of randomly selecting 3 white discs in a row, without replacement, is $\frac{1}{6}$. Find the number of white discs in the urn.

$$P(3 \text{ white}) = P(2 \text{ white}) \times P(3\text{rd disc is white})$$

$$\frac{1}{6} = \frac{1}{3} \times \frac{w-2}{b+w-2}$$

$$1 = 2 \times \frac{w-2}{b+w-2}$$

$$2w - 4 = b + w - 2$$

$$b = w - 2 \quad \cdots \quad \cdots \quad \text{(2)}$$

Substitute into (1):

$$2w^2 - 2w = (w-2)^2 + 2w(w-2) - (w-2)$$

$$2w^2 - 2w = w^2 - 4w + 4 + 2w^2 - 4w - w + 2$$

$$0 = w^2 - 7w + 6$$

$$0 = (w-1)(w-6)$$

$$w = 1, 6$$

But $w \geq 3$, $\therefore w = 6$