» Probability | Diagnostic Quiz

TIME ALLOWED: **40 MINUTES**

MARKS

1. A man finds that he is late for work on 10% of occasions if he is on time the previous day, and on 20% of occasions if he is late the previous day. If he was on time on Monday, what is the probability that he will be on time on Wednesday? Illustrate with a tree diagram. **2**

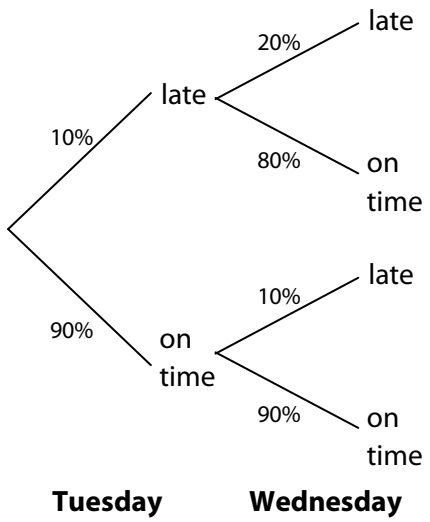
2. To gain a driver's licence in NSW, both a written test and a practical driving test must be passed. Statistics show that 70% pass the written test on the first attempt, while 90% of those who sit the test a second time will pass. 60% pass their first practical test and 80% pass their second practical test. Suppose the written and practical test are independent.
   Calculate, as a percentage, the probability of:
   a. Passing the written test on the second attempt **1**
   b. Passing the written test after no more than two attempts **1**
   c. Requiring a third written test **1**
   d. Passing the practical test on the second attempt **1**
   e. Receiving a licence after two written tests and one practical test **1**

3. A hand of three cards is dealt from a pack of 52 playing cards.
   Calculate, as a fraction, the probability of:
   a. Receiving the ace of hearts **2**
   b. All three cards being spades **2**
   c. Being dealt at least one diamond **2**

4. An urn contains w white discs and b black discs. The probability of randomly selecting 2 white discs in a row, without replacement, is $\frac{1}{3}$.
   a. Write an equation that shows the relationship between w and b. **1**
   b. The probability of randomly selecting 3 white discs in a row, without replacement, is $\frac{1}{6}$. **3**
   Find the number of white discs in the urn.

» Answers

1.



A man finds that he is late for work on 10% of occasions if he is on time the previous day, and on 20% of occasions if he is late the previous day. If he was on time on Monday, what is the probability that he will be on time on Wednesday? Illustrate with a tree diagram.

$$\begin{aligned} P(\text{Wednesday on time}) &= P(\text{on time, on time}) \\ &\quad + P(\text{late, on time}) \\ &= (90\%)^2 + (10\% \times 80\%) \\ &= 81\% + 8\% \\ &= 89\% \end{aligned}$$

2. To gain a driver's licence in NSW, both a written test and a practical driving test must be passed. Statistics show that 70% pass the written test on the first attempt, while 90% of those who sit the test a second time will pass. 60% pass their first practical test and 80% pass their second practical test. Suppose the written and practical test are independent.

a. $$\begin{aligned} P(\text{pass written test, 2nd attempt}) &= P(\text{fail 1st written test}) \times P(\text{pass 2nd written test}) \\ &= 30\% \times 90\% \\ &= 27\% \end{aligned}$$

b. $$\begin{aligned} P(\text{pass written test, } \leq 2 \text{ attempts}) &= P(\text{pass 1st attempt}) + P(\text{pass 2nd attempt}) \\ &= 70\% + 27\% \\ &= 97\% \end{aligned}$$

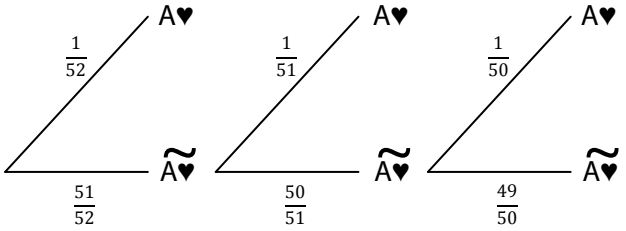
c. $$\begin{aligned} P(\text{require 3rd written test}) &= 1 - P(\text{pass written test, } \leq 2 \text{ attempts}) \\ &= 3\% \end{aligned}$$

d. $$\begin{aligned} P(\text{pass practical test, 2nd attempt}) &= P(\text{fail 1st practical}) \times P(\text{pass 2nd practical}) \\ &= 40\% \times 80\% \\ &= 32\% \end{aligned}$$

e. $$\begin{aligned} P(\text{pass 2nd written , 1st practical}) &= 27\% \times 60\% \\ &= 16.2\% \end{aligned}$$

3. A hand of three cards is dealt from a pack of 52 playing cards.

a. Receiving the ace of hearts



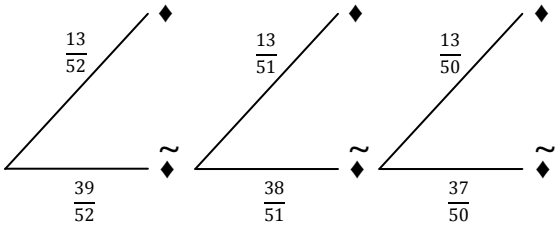
1st **2nd** **3rd**

$$\begin{aligned} P(\text{ace of hearts}) &= P(\text{received 1st}) + P(\text{received 2nd}) + P(\text{received 3rd}) \\ &= \frac{1}{52} + \left(\frac{51}{52} \times \frac{1}{51}\right) + \left(\frac{51}{52} \times \frac{50}{51} \times \frac{1}{50}\right) \\ &= \frac{1}{52} + \frac{1}{52} + \frac{1}{52} \\ &= \frac{3}{52} \end{aligned}$$

b. All three cards being spades

$$\begin{aligned} P(3 \text{ spades}) &= P(1\text{st spade}) \times P(2\text{nd spade}) \times P(3\text{rd spade}) \\ &= \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \\ &= \frac{11}{850} \end{aligned}$$

c. Being dealt at least one diamond



1st **2nd** **3rd**

$$\begin{aligned} P(\geq 1 \text{ diamond}) &= 1 - P(\text{no diamonds}) \\ &= 1 - \left(\frac{39}{52} \times \frac{38}{51} \times \frac{37}{50}\right) \\ &= \frac{997}{1700} \end{aligned}$$

4. An urn contains w white discs and b black discs. The probability of randomly selecting 2 white discs in a row, without replacement, is $\frac{1}{3}$.

   a. Write an equation that shows the relationship between w and b.

$$\begin{aligned} P(\text{2 white}) &= P(\text{1st disc is white}) \times P(\text{2nd disc is white}) \\ \frac{1}{3} &= \frac{w}{b+w} \times \frac{w-1}{b+w-1} \\ 3w(w-1) &= (b+w)(b+w-1) \\ 3w^2 - 3w &= b^2 + bw - b + bw + w^2 - w \\ 2w^2 - 2w &= b^2 + 2bw - b \quad \cdots\cdots ① \end{aligned}$$

   b. The probability of randomly selecting 3 white discs in a row, without replacement, is $\frac{1}{6}$. Find the number of white discs in the urn.

$$\begin{aligned} P(\text{3 white}) &= P(\text{2 white}) \times P(\text{3rd disc is white}) \\ \frac{1}{6} &= \frac{1}{3} \times \frac{w-2}{b+w-2} \\ 1 &= 2 \times \frac{w-2}{b+w-2} \\ 2w - 4 &= b + w - 2 \\ b &= w - 2 \quad \cdots\cdots ② \end{aligned}$$

   Substitute into ①:

$$\begin{aligned} 2w^2 - 2w &= (w-2)^2 + 2w(w-2) - (w-2) \\ 2w^2 - 2w &= w^2 - 4w + 4 + 2w^2 - 4w - w + 2 \\ 0 &= w^2 - 7w + 6 \\ 0 &= (w-1)(w-6) \\ w &= 1,\ 6 \end{aligned}$$

   But $w \geq 3,\ \therefore w = 6$