# » Gradients of Perpendicular Lines

In co-ordinate geometry, the term *gradient* refers to the slope or steepness of a straight line. If a line has a gradient $m = 1$, that means that for every unit we travel in the positive x-direction, the line travels 1 unit in the positive y-direction. For gradient $m = 2\frac{1}{2}$, the line travels $2\frac{1}{2}$ units in the positive y-direction for every unit in the positive x-direction.

When lines are perpendicular, their gradients are related. It only takes a few simple geometric observations to reveal the nature of this relationship. Consider two lines (l_1 and l_2) that we know to be perpendicular, whose gradients are m_1 and m_2. Both lines pass through the origin.
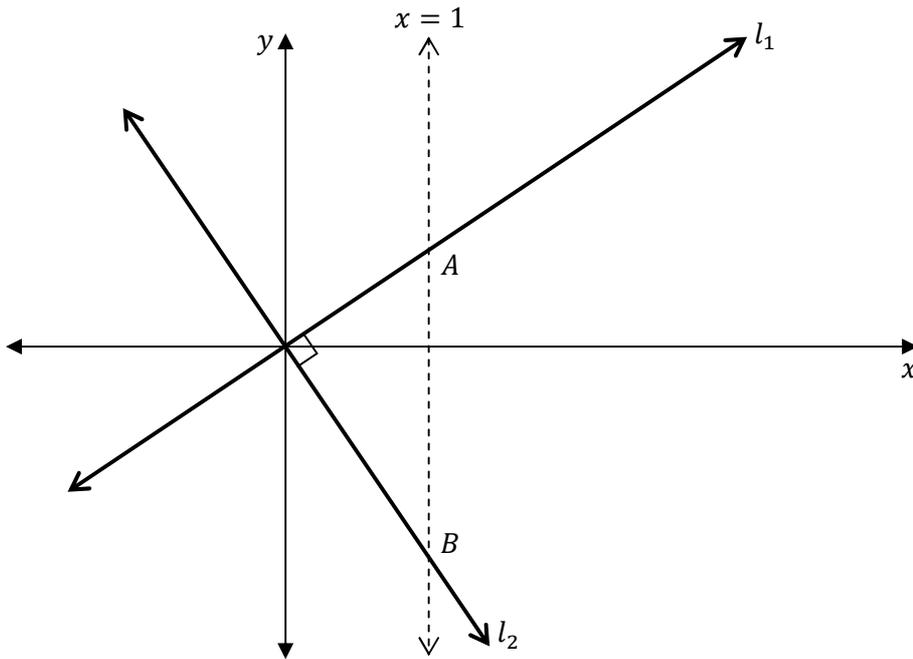
» Questions

1. What are the equations of lines l_1 and l_2?
2. Lines l_1 and l_2 intersect with the line $x = 1$ at points A and B respectively. What are the co-ordinates of these two points?
3. If the origin is point O, then calculate the distances OA, OB and AB.
4. Form an equation that combines OA, OB and AB using Pythagoras' theorem.
5. Simplify this equation, making m_1 the subject, and thereby explain the relationship between the gradients of two lines that are perpendicular.

» Solutions

1. What are the equations of lines l_1 and l_2?

The equation of line l_1 is $y = m_1 x$; the equation of line l_2 is $y = m_2 x$.

2. Lines l_1 and l_2 intersect with the line $x = 1$ at points A and B respectively. What are the co-ordinates of these two points?

$A(1, m_1)$

$B(1, m_2)$

3. If the origin is point O, then calculate the distances OA, OB and AB.

Using the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, the distances are:

$OA = \sqrt{1 + (m_1)^2}$

$OB = \sqrt{1 + (m_2)^2}$

$AB = m_1 - m_2$

4. Form an equation that combines OA, OB and AB using Pythagoras' theorem.

$OA^2 + OB^2 = AB^2$

$\therefore 1 + (m_1)^2 + 1 + (m_2)^2 = (m_1 - m_2)^2$

5. Simplify this equation, making m_1 the subject, and thereby explain the relationship between the gradients of two lines that are perpendicular.

$2 + (m_1)^2 + (m_2)^2 = (m_1)^2 - 2m_1 m_2 + (m_2)^2$

$$2 = -2m_1 m_2$$

$$m_1 m_2 = -1$$

$$m_1 = \frac{-1}{m_2}$$

$\therefore$ When two lines are perpendicular, their gradients are *negative reciprocals of each other* (that is, they multiply together to give -1).